2014 III 13 | 1000 | Seat No. [ ][ ][ ][ ][ ]

Time: 2½ Hours **MATHEMATICS**

Subject Code

H 7 5 4

R

Total No. Of Questions: 8 **(Printed Pages: 7)** **Maximum Marks: 80**

INSTRUCTIONS:

(i) The question paper consists of eight main questions and each question contains four sub-questions.

(ii) All the questions are compulsory.

(iii) Answer each question on a fresh page.

(iv) Figures to the right indicate maximum marks.

(v) Use of calculator is not allowed.

Q.1 (A) Select and write the correct alternative from the choices given below **[1]**

If $A = \begin{bmatrix} 0 & 2 & x \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{bmatrix}$ is a skew – symmetric matrix,

then the value of x is ________

- 2
- 1
- 0
- -1

(B) If the points (a,b), (c,d) and (a-c, b-d) are collinear then using the properties of determinants prove that $ad = bc$ [2]

(C) Given the matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix}$ find the value of 'a' and 'b' such that $A^2 + aA + bI = O$, where I is the Identity matrix of order 2.

Hence, find A^{-1} [3]

(D) Define the continuity of a function f(x) at a point x = a on its domain. [4]

The function f(x) defined by

$$f(x) = \begin{cases} \dfrac{\sin x \cdot \log(1+Ax)}{2x^2} & , x<0 \\ \dfrac{1}{2} & , x=0 \\ \dfrac{1-\sqrt{\cos Bx}}{2\ Bx^2} & , x>0 \end{cases}$$

is continuous at the point x = 0. Find the values of A and B.

Q.2 (A) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $A + A' = I$, [1]

Where A' denotes the transpose of matrix A and I is the Identify matrix of order 2, find the value of θ.

(B) Find the value of [2]

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

(C) Using the properties of determinants, prove that [3]

$$\begin{vmatrix} x & x^2 & 1+\lambda x^3 \\ y & y^2 & 1+\lambda y^3 \\ z & z^2 & 1+\lambda z^3 \end{vmatrix} = (1+\lambda xyz)(x-y)(y-z)(z-x)$$

For any scalar λ.

(D) If $x\sqrt{2+y} + y\sqrt{2+x} = 0$, prove that [4]

$$\frac{dy}{dx} = \frac{-4}{(x+2)^2}$$

Q.3 (A) Select and write the correct alternative from the choices given below [1]

If $f: A \rightarrow B$ is a bijective function and $g: B \rightarrow A$ is the inverse of f, then fog is equal to ________

- I_A
- g
- f
- I_B

(B) If $y = \sec(\tan^{-1} x)$, find $\frac{dy}{dx}$ at $x = 1$ [2]

(C) Show that the function $f: R \rightarrow R$ defined by $f(x) = 3x^3 + 7$ is one – to – one and onto for all $x \in R$ [3]

(D) Find $\int_1^3 (2x+1)dx$ by expressing the integral as the limit of a sum. [4]

Q.4 (A) Select and write the correct alternative from the choices given below [1]

If $\bar{a} \perp \bar{b}$ and $(\bar{a} + \bar{b}) \perp (\bar{a} + m\bar{b})$ then $m =$ ______

- $\dfrac{|\bar{a}|^2}{|\bar{b}|^2}$
- $\dfrac{|\bar{b}|^2}{|\bar{a}|^2}$
- 1
- $|\bar{a}|^2$

(B) Using vectors, find the area of the triangle with vertices A (2, -3, 4), B (1, -2, 3) and C (0, -2, 1) [2]

(C) If $\bar{a}, \bar{b}$ and $\bar{c}$ are three vectors then prove that [3]

$[\bar{a} - 2\bar{b} \quad \bar{b} - 2\bar{c} \quad \bar{c} - 2\bar{a}] = -7[\bar{a} \quad \bar{b} \quad \bar{c}]$

(D) A chemical industry produces two fertilizers A and B. Two ingredients C and D are used in the two fertilizers. The following table gives the units of ingredients C and D (per kg) of fertilizers A and B as well as minimum requirements of C and D and costs per kg of A and B. [4]

	Fertilizers	in units	minimum requirement
	A	B	(in units)
Ingredient C (per kg)	2	3	50
Ingredient D (per kg)	4	2	44
Cost per kg (in Rupees)	20	40	

Find the quantities of A and B which would minimize the cost.

Also, find the minimum cost, by formulating a linear programming problem and solving it graphically.

Q.5 (A) Select and write the correct alternative from the choices given below [1]

$$\int \frac{2^x}{\sqrt{1-4^x}}\,dx = \_\_\_\_\_\_$$

- $\frac{1}{4}\sin^{-1}(2^x) + c$
- $\frac{1}{\log 2}\sin^{-1}(2^x) + c$
- $(\log 2)\sin^{-1}(2^x) + c$
- $\frac{1}{2}\sin^{-1}(2^x) + c$

(B) Prove that [2]

$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$ for $x \in R$

(C) Evaluate [3]

$$\int_0^{\pi/2} e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

(D) Attempt **any one** of the following: [4]

(i) Find $\int \frac{dx}{\sin x(5-4\cos x)}$

(ii) Find $\int (\tan^{-1} x)^2 \cdot x \; dx$

Q.6 (A) Find the Cartesian equation of the line passing through the point (1, -2, 5) and perpendicular to the plane $x+y-z=6$ [1]

(B) Find the rate of change of volume of the sphere with respect to its surfaoe area when its radius is 2 units. [2]

(C) Prove that [3]

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

(D) A window is in the form of a semicircle above the rectangle. If the perimeter of the window is p cms, show that the window will allow the maximum possible light when the radius of the semi-circle is $\frac{p}{\pi + 4}$ cms. [4]

Q.7 (A) A drawer contains 3 red and 5 blue socks which are well mixed. If 2 socks are pulled out at random without replacement then find the probability that both the socks are of the same colour. [1]

(B) Solve the following differential equation [2]

$$\frac{dy}{dx} = xy + x + y + 1$$

(C) Attempt **any one** of the following [3]

(i) Find the equation of the plane which passes through the points (1, 0, 0) and (0, 1, 0) and makes an angle $\pi/4$ with the plane x+y = 3

ii) If $\overline{n}$ is a vector of magnitude $2\sqrt{3}$ units and makes equal acute angles with the positive directions of all the three coordinate axes, find the vector equation of the plane passing through the point (1, -1, 2) and perpendicular to the vector $\overline{n}$. Write vector $\overline{n}$ in its component form.

(D) Find the particular solution for the differential equation [4]

$$(x^2 + 1)\frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1)\cos x$$

Where y = 0 when x = 0

Q.8 (A) State the multiplication theorem on probability [1]

(B) If the scalar product of the vector $\bar{a} = 2\hat{j} - \hat{k}$ and the unit vector along the vector $\hat{i} + m\hat{j} - 2\hat{k}$ is equal to 2 then find 'm'. [2]

(C) Prove that [3]

$$\int_0^{2a} f(x)\,dx = \int_0^{a} [f(2a-x) + f(x)]\,dx$$

(D) Attempt **any one** of the following [4]

(i) A biased die is twice as likely to show an even number as an odd number. The die is rolled three times. If the occurence of an even number is considered a success, then find the probability distribution of number of successes. Also, find the mean number of successes.

(ii) A die is tossed twice and getting 'a number greater than 4' is considered a success. Find the variance and the probability distribution of the number of successes.

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