2015 III 13 **1000** **Seat No.** | | | | | |

Time: 2 ½ Hours **MATHEMATICS**

Subject Code

H	7	5	4

C

Total No. Of Questions: 07 **(Printed Pages: 07)** **Maximum Marks: 80**

INSTRUCTIONS:

(i) ***This question paper contains seven main questions.***

(ii) ***All seven questions are compulsory.***

(iii) ***Answer each main question on a fresh page.***

(iv) ***Use of calculator is not allowed.***

(v) ***Log tables will be supplied on request.***

(vi) ***Graphs should be drawn on the answer paper only.***

(vii) ***For each main questions the sub-questions carry the following marks.***

A=1 mark, B=2 marks, C=3 marks, D=4 marks, E=5 marks.

Q. 1 (A) Select and write the correct alternative from the choices given below. **[1]**

A diagonal matrix in which all the diagonals elements are equal is called ________

- **Row matrix**
- **Column matrix**
- **Scalar matrix**
- **Rectangular matrix**

(B) Using determinants find the area of the triangle with vertices (2, 1), (1, - 4) and (5, 6) **[2]**

(C) Let A = N × N and '*' be a binary operation on A defined by **[3]**

(a, b) * (c, d) = (a + c, b + d).

(i) Show that '*' is commutative.

(ii) Find the identity element for '*' on A if it exists.

(D) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 2 & 3 \\ 6 & 1 & 5 \end{bmatrix}$ **find** A^{-1} **using adjoint of the matrix.** **[4**

Q. 2 (A) Select and write the correct alternative from the choices given below. **[**

If $\frac{2}{\sqrt{x}} = \frac{1}{y}$ **then** $\frac{dy}{dx}$ **at x = 1 is ________**

- $\frac{1}{4}$
- 2
- $\frac{1}{2}$
- 4

(B) If $x = (t^2 + 1)^3$ and $y = (2t^2 - 5)^2$, find $\frac{dy}{dx}$ [2]

(C) If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$ then find the value of $\frac{p}{x} + \frac{q}{y} + \frac{r}{z}$ using properties of determinants. [3]

(D) A function f is defined by [4]

$$f(x) = \frac{\log(1+3x)}{x} - B; \text{ for } x < 0$$

$$= \frac{x^2 + 2x}{x+1} + 5 \text{ ; for } x = 0$$

$$= \frac{e^{\text{SinAx}} - 1}{x} \text{ ; for } x > 0$$

If f is continuous at x = 0, find the values of constants A and B.

Q. 3 (A) Select and write the correct alternative from the choices given below. [1]

A function f is said to be invertible if and only if f is ....

- only onto but not one-one.
- both one-one and onto.
- only one-one but not onto.
- neither one-one nor onto.

(B) Find the point on the curve $y = 3x^2 - 12x + 6$ at which the tangent is parallel to the X – axis and hence find the equation of the tangent at that point. [2]

(C) If $\bar{a} = \hat{i} - \hat{j} + \hat{k}$ and $\hat{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ then find, [3]

(i) Sine of the angle between $\bar{a}$ and $\bar{b}$

(ii) Unit vectors perpendicular to $\bar{a}$ and $\bar{b}$

(D) Using integration prove that [4]

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left| x + \sqrt{x^2 + a^2} \right| + c$$

Q. 4 (A) Select and write the correct alternative from the choices given below. [1]

$$\int_0^{\pi/2} \frac{1 - \cos x}{1 + \cos x} dx \text{ is } \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

- $2 - \frac{\pi}{2}$
- $2 + \pi$
- $2 + \frac{\pi}{2}$
- $2 - \pi$

(B) If $y = x^{\tan x} + e^{\log x}$, find $\frac{dy}{dx}$ [2]

(C) Find the shortest distance between the lines whose equations are [3]

$$\bar{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k} \text{ and } \bar{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

(D) Prove that $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if f (x) is an even function [4]

Q. 5 (A) If $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j}$ and $\bar{c} = -\bar{j} + \bar{k}$ then find $[\bar{b}\ \bar{c}\ \bar{a}]$ [1]

(B) If $\bar{a}$ is a unit vector and $(\bar{x}+\bar{a})\bullet(\bar{x}-\bar{a}) = 12$ then find $|\ \bar{x}\ |$. [2]

(C) Find the equation of the plane, passing through the points (1, 1, 2) and (2, -2, 2) and perpendicular to the plane 6x - 2y + 2z = 9 [3]

(D) Solve the following linear programming problem graphically. [4]

Maximise $Z = 5x + 7y$

Subject to the constraints: :

$x + y \leq 4$

$2x + y \leq 8$

$3x + 8y \leq 24$

$x \geq 0$ and $y \geq 0$

Q. 6 (A) If A and B are two independent events of a sample space and P(A) = 0.3, P(B) = [illegible].
Find P(A and not B). [1]

(B) Form the differential equation representing the family of curves $y^2 = a(b^2 - x^2)$ where a and b are arbitrary constants. [2]

(C) In a store room there are 1000 orange juice bottles, 2000 apple juice bottles and 3000 mango juice bottles. The probability that the bottle of orange juice, apple juice and mango juice breaks is 0.3, 0.2 and 0.05 respectively. One of the bottle breaks, find the probability that it is an apple juice bottle. [3]

(D) Attempt **any one** of the following: [4]

(i) Solve the differential equation

$$(x - y)\frac{dy}{dx} = x + 3y$$

(ii) Solve the differential equation.

$(1 + y^2)\ (1 + \log x)\ dx + x\ dy = 0$. Also find the particular solution given that

$y = 1$ when $x = 1$.

(E) Attempt **<u>any one</u>** of the following: [5]

(i) Find $\int \frac{\tan\theta + \tan^3\theta}{1 + \tan^3\theta}\, d\theta$

(ii) Find $\int \cos 2x \log(3 + 4\tan x)\, dx$

Q. 7 (A) Find the value of $\cot(\tan^{-1} a + \cot^{-1} a)$ [1]

(B) A random variable X has the following probability distribution: [2]

X	0	1	2
P(X)	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

Find the mean and variance of X.

(C) Prove that $\cot^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}\right) = \frac{1}{2}\mathrm{Sin}^{-1}x,\ x\ \varepsilon\ \left(0, \Pi/4\right)$ [3]

(D) Find $\int_0^2 (x^2+1)\,dx$ as the limit of a sum. [4]

(E) Attempt **any one** of the following [5]

(i) The perimeter of an isosceles triangle is 100cms. If its base is changing at the rate of 4cms/sec, find the rate at which its altitude is changing when the base is 30cms.

(ii) An open box with a square base is to be made out of a given quantity of cardboard of area 27 square units. Find the dimensions of the box so that the volume of the box is maximum.
